Form Factors of Branch Point Twist Fields: Exercises

1. Going back to the lecture notes, show that if the S-matrix has an integral representation of the form,
$$S_{ab}(\theta) = \exp\left[\int_0^\infty \frac{dt}{t} f_{ab}(t) \sin\frac{\theta t}{i\pi}\right],$$
then the function
$$F_{\mathbf{min}}^{\mathcal{T}|ab}(\theta) = \exp\left[\int_0^\infty \frac{dt}{t}\frac{f_{ab}(t)}{\sinh(nt)} \sin^2\left(\frac{it}{2}\left(n+\frac{i\theta}{\pi}\right)\right)\right]$$
satisfies
$$F_{\mathbf{min}}^{\mathcal{T}|ab}(\theta) = S_{ab}(\theta) F_{\mathbf{min}}^{\mathcal{T}|ba}(-\theta) = F_{\mathbf{min}}^{\mathcal{T}|ba}(-\theta + 2\pi i n),$$
Assume parity invariance, that is $S_{ab}(\theta) = S_{ba}(\theta)$.

2. Show that the two-particle form factor of particles in the same copy
$$F_2^{\mathcal{T}|ab}(\theta) = \frac{\langle \mathcal{T}\rangle \sin\frac{\pi}{n}}{2n\sinh\left(\frac{i\pi-\theta}{2n}\right)\sinh\left(\frac{i\pi+\theta}{2n}\right)} \frac{F_{\mathbf{min}}^{\mathcal{T}|ab}(\theta)}{F_{\mathbf{min}}^{\mathcal{T}|ab}(i\pi)}$$
satisfies Watson's equations as well as the (first) kinematic residue equation:
$$-i\lim_{\theta\to i\pi}(\theta - i\pi) F_2^{\mathcal{T}|ab}(\theta) = F_0^{\mathcal{T}} := \langle \mathcal{T}\rangle$$

3. Prove the identities given in the penultimate page of Lecture 2B.

4. By computing the analytic continuation in n of $h(\theta, n)$, compute
$$\lim_{n\to 1}\frac{\partial h}{\partial n} \quad \text{with} \quad h(\theta,n) := \sum_{i=1}^n \sum_{j=1}^n h_{ij}(\theta, n)$$
where $h_{ij}(\theta) := F_2^{\mathcal{T}|(a,i)(a,j)}(\theta)$. Check your result numerically for the free massive boson and fermion which have $F_{\mathbf{min}}^{\mathcal{T}|11}(\theta) = 1$ and $F_{\mathbf{min}}^{\mathcal{T}|11}(\theta) = -i\sinh\frac{\theta}{2n}$, respectively for particles in the same copy (note that there is a single particle species in these theories). Hint: Use the cotangent trick as we saw in Lecture 3A.